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Upper Limit for Number Density of Comets Near Pluto

Abstract:

The maximum number density of the comet cloud in the outer solar system was calculated by the effect comets might have on Pluto and its moon Charon. If there is a high enough density of comets, the orbit of the moon can be altered in such a way that allows us to rule out that respective density of comets. These results then can be applied to the density of comet objects during the formation of the Oort cloud and Kuiper belt. The program runs with variable density of the comet cloud through which Pluto travels. In the computer simulation, the distance between Pluto and its moon is monitored for drastic changes when tested with different size comets. The computer simulation shows a fifty percent change in orbital radius of Charon about Pluto when the number of Kuiper belt objects is 350 times above the present estimated number. A total mass of 17 M_{earth} would give this resulting change in orbital radius. The data also shows that the current estimated number of comets do not detectably affect Charon's orbit.

Introduction:

In 1930, Pluto was discovered due to a prediction of a ninth planet based on incorrect data of Uranus' motion. Even though it was almost immediately known that Pluto was extremely small in mass compared to the rest of the planets and was not affecting Uranus' orbit, it was left classified as a planet until 2006 when the IAU defined strict rules for the classification of a planet. Since Pluto's orbit crosses with Neptune's, Pluto was reclassified as a dwarf planet. Kuiper belt objects were not proposed to exist until later in the 20th century, as a result Pluto's importance was not known until the discovery of the next object. David C. Jewitt and Jane Luu[6] finally discovered the next Kuiper Belt Object in 1992 know as 1992QB1. These objects range from comets to small dwarf planets in a distance of 30 to 50 AU away from the sun. At about 39.4 AU there is a 2:3 resonance with the orbit of Neptune that collects many comets and is also near Pluto's orbit. By observation of Pluto and its satellites[9], the ratio of Charon's to Pluto's mass is 0.11 and also the Pluto-Charon system is shown to have a total mass of about 1.47×10^{22}kg. Charon is also found to have a radius of 600 kilometers by stellar occultation observations[10] and Pluto is close to 1150 kilometers in radius. Charon's orbit is known[12] to have a semi-major axis of 19,571 ± 4 km, and have an eccentricity of 0.00000 ± 0.00007. Occultation refers to the amount of light that is blocked when one object passes in front of another object. This measurement of Charon's orbit corresponds to about .002 arcseconds of precision as seen from earth, showing that Charon's orbit is very well known. The orbital period of Pluto is 6.3872304 ± 0.0000011 days.

Although the orbits of these objects in the outer solar system such as Pluto and Charon are known to many significant figures, one issue with the information is that people have only been observing them for less than the last one hundred years. This means that changes that could be occurring slowly over time to the orbits are not known at all. Although one can assume that since the solar system has been around for billions of years it is unlikely to be undergoing any major changes, the only way to know something about the solar system for sure is to observe it over a very long period of time.

This is also a problem because the lifespan of people is very much shorter than length of time needed to observe the outer solar system. Since the technology to observe these distant objects is very new, there will not be very old records to find changes until very far into the future. Stellar occultation observations are also affected by diffraction and the orientation of objects that pass in front of a star, so it is difficult to get exact results for comets whose occultation properties are not very well known. The distribution of larger comets is known to go as a power law that is proportional to the comet's size, it goes as its radius to the negative fourth power. There is also a sharp cliff[2] in the number density of comets at 50 AU away from the sun where outside this distance the number density drops significantly. Although the comet density is by all means not an even distribution, the net effect of gravity from the entire belt of comets has little effect on Charon because Pluto is in the same potential from it also.

The main cause of the orbit disturbance between Pluto and Charon is due to very nearby interactions that are very strong due to the force being r^{-2} dependent where r is the distance between the objects. When a comet passes closer to Charon than Pluto, it feels a very strong force relative to the effect on Pluto. Objects in the Kuiper belt do not have well know physical attributes because the only measurement that can be taken is the amount of light that they reflect back to Earth. Not only does this make it difficult to have accurate data on the comets, in addition to their far distance from the earth, they reflect very little light. The main measured property of comets is the proportion of the incoming light to the reflected intensity, which is called the albedo. The albedo for all comets in the Kuiper Belt is assumed to be .04 based on comet nuclei, this means that they reflect only 4% of the incoming light[1]. There is likely to be a large source of error associated with using the same albedo for all comets, some may have lots or very few ice patches that affect how much light is reflected back to Earth. Since the actual mass density is not well known, this C++ code developed for the present work models the effect of gravity between Pluto's moon Charon and the comets in the nearby area. By calculating the force between the objects $F_i = m_i a = GMm_i/R^2$, the code then sums up all forces and continuously calculates new positions and velocities for all bodies in the system. Since the semi-major axis and eccentricity for Charon are know to .04 and .007 percent respectively, a change of fifty percent would be substantial. The program outputs the distance between Pluto and Charon, if the distance changes by fifty percent it would correspond to an amount of force from comets that would definitely not be able to result from current estimated number of comets. This means there is too much mass density of comets, therefore in the program where there is just not enough mass to alter the orbit of Charon by half of its current orbital radius would be the very upper limit to the comet density in the area around Pluto.

<u>Theory</u>:

When two objects in space interact through gravity, there is a transfer of energy and momentum that alters the course of both objects. By using Charon's stable orbit around Pluto, it is know that there are not enough objects passing by to disturb it. By creating a computer simulation, the conditions for which Charon's orbit becomes unstable can be found. Since changes in Charon's motion are only dependent on Kuiper Belt Objects that pass by, the maximum density of these objects can be found when Charon's orbit is greatly disturbed. Since the amount of observable data from the outer solar system is very small, this leaves many people to use computer simulations that were not possible before computing power has reached its current speed. Computers can now do billions of calculations per second, which is necessary for integrating differential equations where high accuracy is required and because of loops the total number of operations approaches 10^{13}. Outputting data to files is one of the slowest operations because writing data to a hard disk is very slow compared to the speed of a processor. Another slow operation for computers is finding square roots, which must be calculated constantly in gravity type force problems where the magnitude of three vectors must be found, such as the distance between objects in three-dimensional space. Computing power is still limiting the computation of similar model code where the positions of thousands of particles are analytically calculated, which is what the original code was written to model but edited for this more specialized project. Since the time the code takes to run scales fairly linearly with the number of objects, and the number of iterations in loops in the code. If it takes on the order of one day for an order of ten objects, the same code using one thousand objects would take about one hundred days to complete. While this is physically doable, it is definitely not practical for a small project of this type. As computing power continues to increase by orders of magnitude, these kinds of numerical simulations and calculations will be much more feasible, and more complex models will be able to be created.

The total mass that is estimated in the Kuiper Belt currently is .05 M_{earth} [11]. The number and mass is found by taking visual surveys of sections of the sky. The number is counted then the mass of each is found relating the size of an occultation to the diameter of a comet then the mass an object is estimated from its diameter by assuming all comets have the same mass density. By counting occultation rate and the amount of light blocked to determine the number and size the Kuiper belt is found to have a population density that can be related to the radius, which goes as $N = R^{-4}$ where N is the number of comets and R is their radius that is found by occultation. The amount of objects with radius greater than 100km is estimated to be 30000. This translates to an average number density of objects that is about 0.16 objects per cubic AU. The normalization for the distribution function of radius to number density is also found by setting the integral from 100km to 2000km of R^{-4} equal to 30000. The upper limit of two thousand kilometers was chosen because the value of the integral becomes fairly constant as a function of radius at such a large upper limit. The observed number density and mass of the Kuiper belt can then easily be compared to the calculated limit. One issue is that the distribution of comets in the outer solar system varies as a function of radius from the sun, due to objects falling into resonance orbits with the large planets. During the early solar system this is likely to not be an issue because most of the comets would not have had time to be stuck into resonances or be kicked out of the solar system, and the distribution of comets would be fairly even with respect to distance.

<u>Simulation arrangement</u>:

This project uses Runge-Kutta 4th order integration to solve a “N-body” problem. Since there is no analytical solution to the motion of multiple bodies due to a gravitational force between them, the only way to solve it is by integrating the differential equation of the sum over I and J of $d^2R_I/dt^2 = Gm_J/R_I^2$ which is all the bodies involved. C++ language is used and written in a program called Xcode for Apple OS X. C++ works very well for this code because it is very object oriented, so the different bodies such as Pluto and the comet can be classified as different objects with different properties. These objects are defined in a separate file than the actual code that is the experiment. The code also uses vectors in all of the calculations for position, velocity, and acceleration, which must also be defined in a separate file and included in order for the code to use vector operations, even for simple addition and subtraction. C++ is also a very easy to learn programming language and intuitive to write, especially it's object oriented nature. It is also easy to output data from the program. By using a loop to check how long far the code has progressed, an output command is used that writes the variables that are needed, such as position, to an output text file. By using the loop, it can accurately tell how much time relative to the simulation has passed for Pluto in the program so the radius of Charon's orbit with respect to time may be plotted in Excel. The program also outputs the calculated orbit of Pluto and Charon around the sun and a section of the orbit can be seen in Figure 1.

The program must keep track of the position of Pluto, Charon, and a comet, by solving for their velocities by the force between the 3 objects and the sun. Since the period of Charon around Pluto is only six days, the step size of the integration must be very small to have a stable circle path around Pluto for 500 million years. Since the time it takes to run the code is linearly dependent on the step size, integration length, and number of bodies it is important to maximize the step size and minimize integration length since they are the only real free parameters that can be changed for the codes purpose. Four copies of the code can be run simultaneously with different parameters and take about 6 hours using a dual processor dual-core 3.0Ghz Intel Xeon "Woodcrest" processors with 2Gb of Error Correction Control ram. Every different version of the code must be compiled individually on the computer it is to be run on, this is done by using command lines in a terminal program on the computer. The compilation command would look like "g++ filename.cpp" while being in the directory that the code and its object files are in, then to run the code that has just been compiled one would type ./a.out in the next line. If there are any errors in the code relating to the machine's language the code will not compile and the location of the errors will be displayed so they can be fixed. If there is an error in the way a calculation is carried out however, the only way to catch the problem is to notice if the output of the code is doing something unexpected and search through based on what parts of the code affect the output. Sometimes the error maybe unnoticeable but cause small errors to the calculation, this makes writing computational experiments very difficult. Computer code written by one person is very difficult to understand by anyone and unfortunately this applies even the original person that wrote the code. The best way to get around this issue is to have two different people write two independent versions of a code that output what should be the same result, then differences can be found. Unfortunately because this method of troubleshooting is twice as time consuming it was not done for this code. The code was read through many times however, the code totals to about 650 lines not including empty lines, this is a relatively low number for codes due to the nature of the loops involved in running the code. This short length makes it slightly easier to search through for errors, which was done many time to eliminate any coding problems.

While the code is running it periodically displays the percentage of the code it has ran through in order to see its progress. Since the code can be cancelled anytime without losing any of the preceding data outputs, being able to see how far the code has run at the point of cancellation would be useful in finding the length of simulated time because the progress of the code is proportional to the simulated time. At the end of the code the total real time it takes to run the entire duration of the code is outputted to get an accurate number for how long it takes to run. By starting with a comet mass of zero, the natural motion of Charon can be found to make sure that Charon moves in a circle around Pluto and any changes to its original orbit are due to the comets. The code has Charon interact with a comet using a cross section that is .001 AU^2 and a mean free path of 1000 AU, this corresponds to one comet per cubic AU. The mean free path length also sets up the time scale of the problem. The time it takes per interaction is the mean free path divided by the speed of Pluto, then multiply by the number of interactions for the total time length which gives the total length of time that passes inside the simulation. With this, the mass density depends on the choice of mass of the comet used in the program. Next a variety of runs using different masses for the bypassing comet are done. The code outputs the distance between Charon and Pluto while the code runs, then the radius of Charon's orbit and is plotted as a function of time to see its changes (Fig 2). All the data was plotted using Microsoft Excel by importing the output data file. The data is written into the text file in such a way that it is easy to import into Excel. Each variable is separated by a space and falls into separate columns in Excel, and subsequent data values are entered on the next line and result in the data points go down their respective columns with increasing time.

Results:

The light blue line in Figure 1 corresponds to interactions with comets of mass $1.4\text{x}10^{21}$kg, which is enough to kick Charon away from Pluto over 500 million years. The yellow line (fig 2) corresponds to $1.1\text{x}10^{21}$kg comets and the pink line to $0.35\text{x}10^{21}$kg comets. The dark blue line is corresponding to $.04\text{x}10^{21}$kg mass comets. Charon's mass density is about 1.60 g/cm^3, similar to rock and a bit denser than water. Using the same mass density as Charon, the radius of a given comet can be solved for by its mass by taking the ratio of Charon's radius cubed to Pluto's radius cubed then multiplying by the mass of the binary system. This finds the fraction of Charon's mass to Pluto's mass, and then the total is found by relating it to the systems mass. This means that having one comet of radius 485km per cubic AU is the maximum before Charon is disturbed. Setting the integral from 485 to 2000 of R^{-4}, R being the radius of the comet, equal to 1.0 AU^{-3} gives the normalization for the number density of comets in the Kuiper belt for the curve of the light blue line in Figure 2 to be 3.5 $\text{x}10^8$. The equation for the number density versus radius of the comet corresponding to the light blue line becomes $n = 3.5\text{x}10^8/R^4 km^4 AU^{-3}$, this is the equation displayed as the blue line in figure 3. The pink line (fig 3) is calculated in a similarly, and it turns out to be smaller than the blue line by only a factor of 350. The estimated total mass currently in the Kuiper belt is .05 Earth masses[8]. The total mass corresponding to the maximum number density of comets while keeping Charon in a stable orbit is then 350 times .05 M_{Earth}, giving 17 Earth masses. The dark blue line's (fig 2) number density function is also calculated, and comes out to be $n=0.1\text{x}10^8/R^4 km^4 AU^{-3}$, the distribution is still ten times larger than the observed number density. This means that the actual comets in the outer solar is ten times smaller in number than a distribution of comets that would cause an effect of about a thousandth of the orbits size over millions of years, therefore it would take millions of years to actually see the change and is undetectable to us presently. Since the orbit of Charon is only known to the ten thousandth place, the observed comet distribution is actually affecting Charon by an undetectable amount to observers on Earth even if one were to watch for the lifetime of the solar system.

Conclusion:

Over the course of 500 million years the largest mass density of comets that allows Charon to be affected in a way that its orbit size changes by less than half is about 350 times greater than the current estimated density of the Kuiper Belt. The comet distribution that causes this change corresponds to a total mass of 17 M_{earth} in the outer solar system. Since high comet number density is related to the frequency of interactions, the chance of an object such as Pluto capturing one as a moon also increases. Pluto is likely to have captured its moon earlier during the solar system's formation, because some models of early solar system formation require the initial Kuiper Belt mass to have a total of 10-35 M_{earth} which is on the same order of magnitude as 17 M_{earth}. The results also show that the current observed distribution of comets has no detectible effect on the orbit of Charon around Pluto, therefore interactions that would cause the capture of a moon such as Charon along with other binary systems must have happened during a period of high comet density. The density must have decreased by some effect and left the objects remaining bound in a stable orbit because they stop having strong interactions with comets.

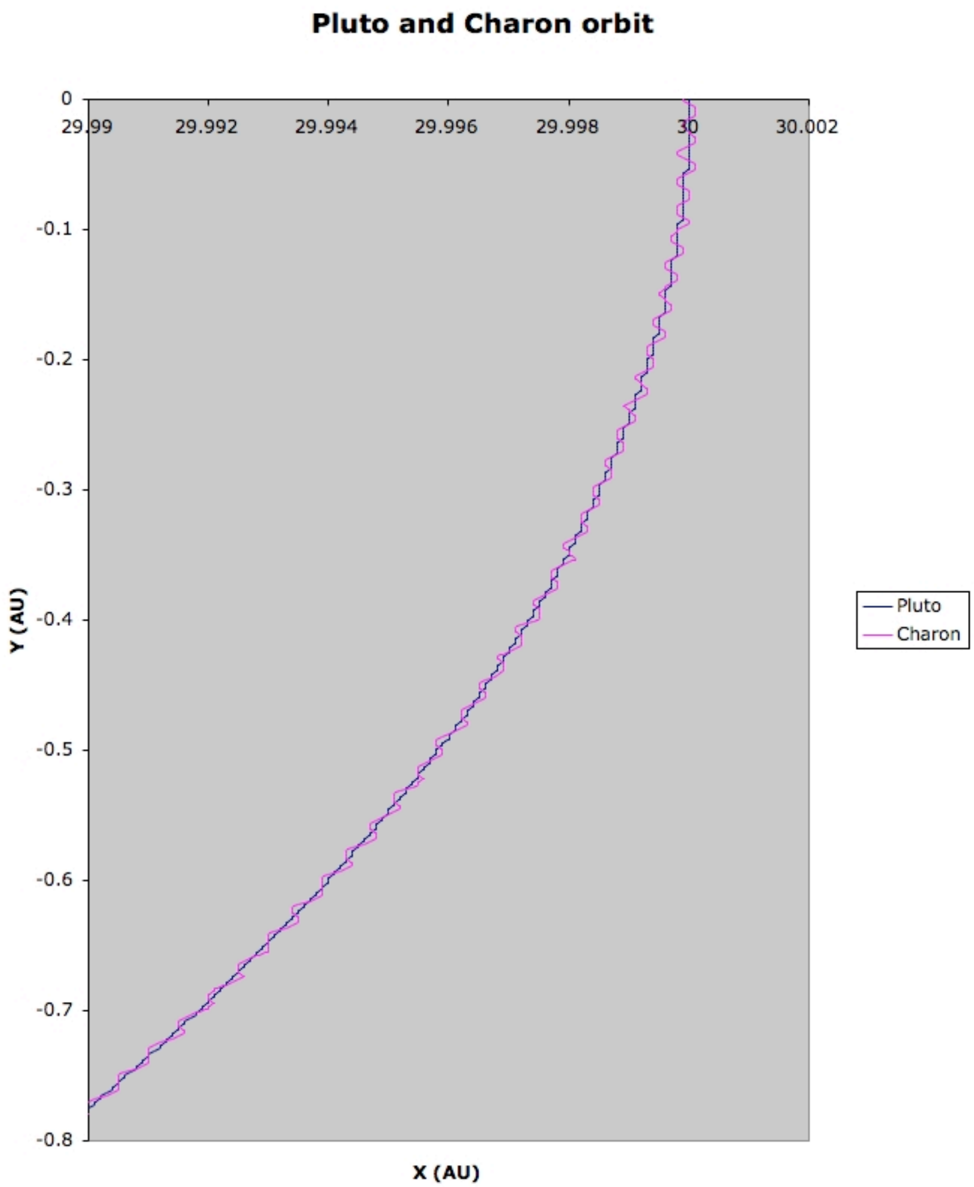

Figure 1 – Small Section of Pluto and Charon's orbit around the sun. The orbit is not smooth due to data being output about every quarter of Charon's orbit. The change in Charon's orbit cannot be seen in a plot such as this one because the timescale for such changes is extremely slow and happens over many orbits of Pluto around the sun. Since Pluto goes around the Sun many times and the coordinates plotted would overlap many times and as such would look like a mess.

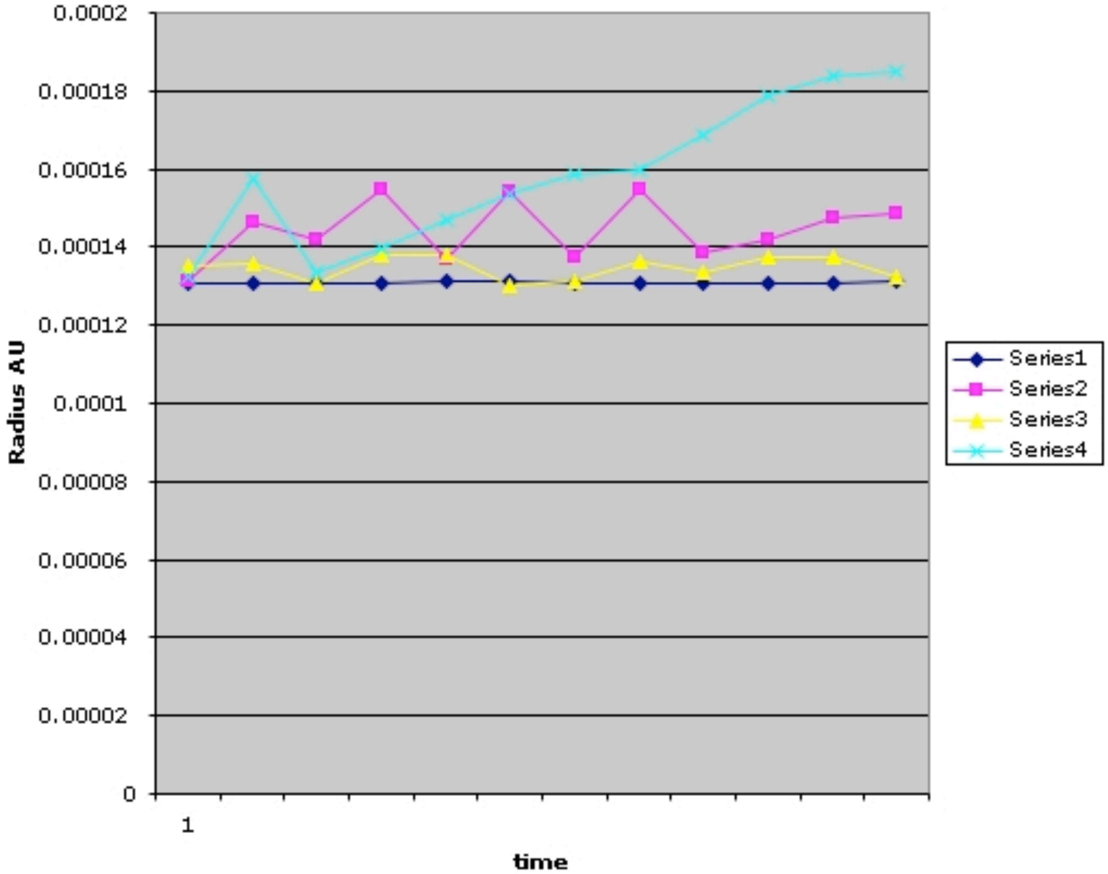

Figure 2 – Radius of Charon's orbit around Pluto. Series number corresponds to increase in density of comet cloud. The light blue line reflects the effect of three and a half times two orders of magnitude more comets than observed. The dark blue line is a density of comets that is still close to an order of magnitude higher than the observed density of comets in the outer solar system, yet has little effect on Charon.

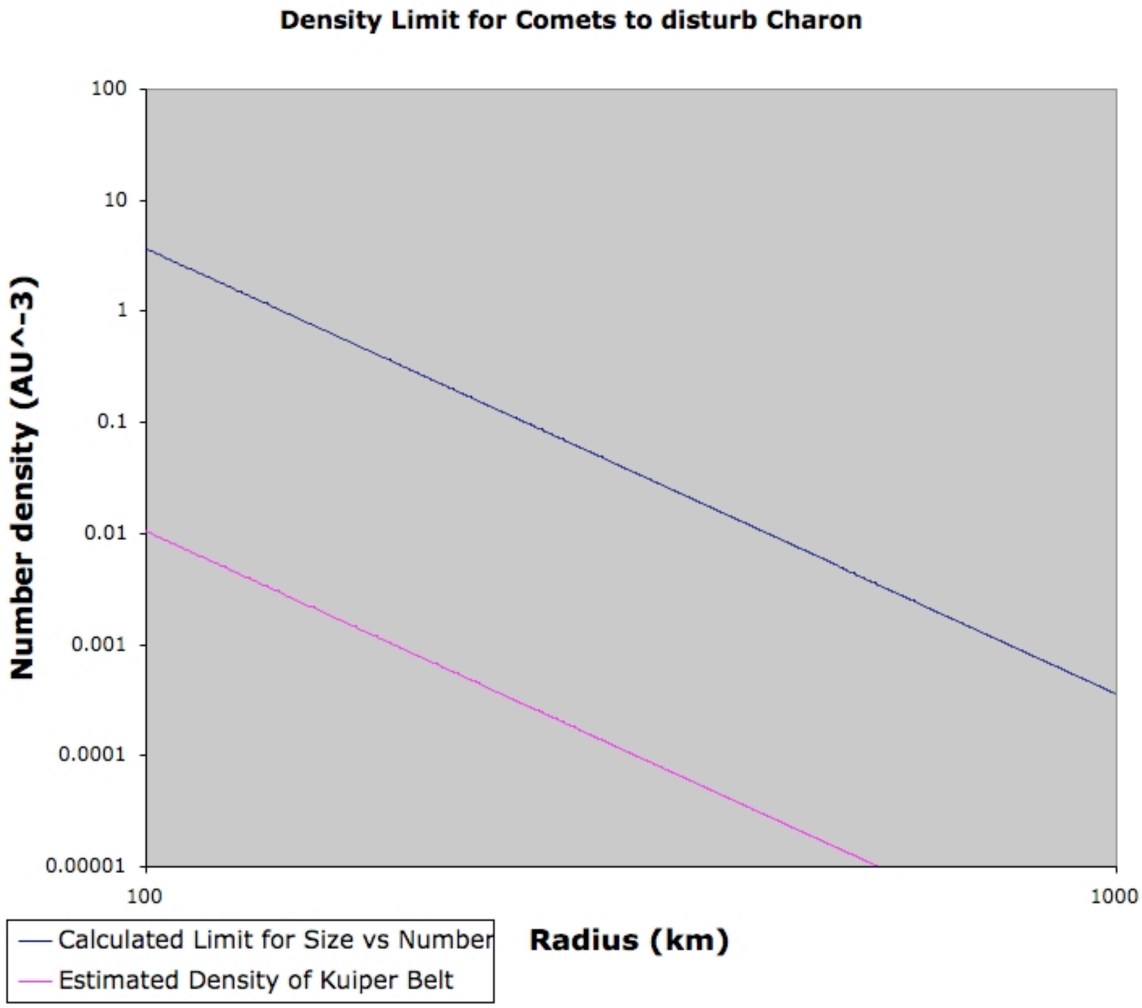

Figure 3 – Each line is a number density function with respect to radius of comets. Distributions of comets below the blue line will not change Charon's orbit by more than 50%, and distributions above it will change the orbit by more than 50% over five hundred million years.

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